5.NF Dividing by One-Half

Alignments to Content Standards

- Alignment: 5.NF.B.7

Tags

• *This task is not yet tagged.*

Solve the four problems below. Which of the following problems can be solved by finding $3 \div \frac{1}{2}$?

a. Shauna buys a three-foot-long sandwich for a party. She then cuts the sandwich into pieces, with each piece being $\frac{1}{2}$ foot long. How many pieces does she get?

b. Phil makes 3 quarts of soup for dinner. His family eats half of the soup for dinner. How many quarts of soup does Phil's family eat for dinner?

c. A pirate finds three pounds of gold. In order to protect his riches, he hides the gold in two treasure chests, with an equal amount of gold in each chest. How many pounds of gold are in each chest?

d. Leo used half of a bag of flour to make bread. If he used 3 cups of flour, how many cups were in the bag to start?
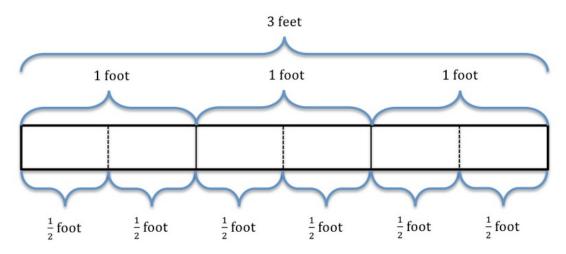
Commentary

This task requires students to recognize both "number of groups unknown" (part (a)) and "group size unknown" (part (d)) division problems in the context of a whole number divided by a unit fraction. It also addresses a common misconception that students have where they confuse dividing by 2 or multiplying by $\frac{1}{2}$ with dividing by $\frac{1}{2}$.

Solutions

Solution: Solution

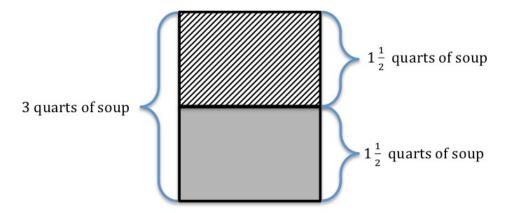$$3 \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = \frac{6}{1} = 6$$

a. If Shauna buys a 3 foot long sandwich and cuts it into pieces that are $\frac{1}{2}$ foot long, her sandwich would look something like the picture below.

3 feet

1 foot   1 foot   1 foot

$\frac{1}{2}$ foot $\frac{1}{2}$ foot $\frac{1}{2}$ foot $\frac{1}{2}$ foot $\frac{1}{2}$ foot $\frac{1}{2}$ foot

By looking at the picture above, we can see that there are 6 pieces that are $\frac{1}{2}$ foot long, which gives the same answer as if we were to compute $3 \div \frac{1}{2}$.

We can also think about this problem without the help of a picture. Shauna has a sandwich that is 3 feet long. If she slices that sandwich into pieces so that each piece is $\frac{1}{2}$ foot long and wants to know how many sandwich slices she would get, Shauna is really asking, "How many groups of $\frac{1}{2}$ are in 3?" which is one of the interpretations of $3 \div \frac{1}{2}$.
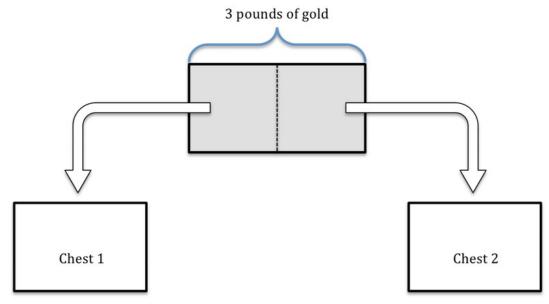
b. If Phil makes 3 quarts of soup for dinner, and his family eats half of the soup for dinner, his situation would look something like the picture below.

1½ quarts of soup

3 quarts of soup

1½ quarts of soup

From this picture above, we can see that Phil's family ate $1\frac{1}{2}$ quarts of soup. This problem cannot be modeled by $3 \div \frac{1}{2}$.
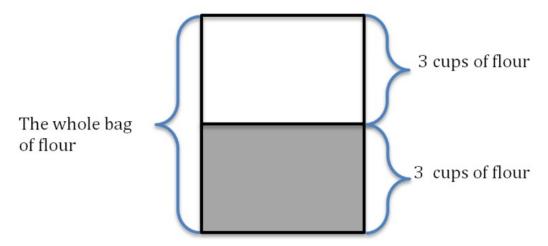
We can also think about this problem without the help of a picture. If Phil makes 3 quarts of soup for dinner, and his family eats $\frac{1}{2}$ of the soup he made, then his family really ate $\frac{1}{2}$ of the 3 quarts of soup. This is the meaning of $\frac{1}{2} \times 3 = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$ not $3 \div \frac{1}{2} = 6$.

c. If a pirate finds 3 pounds of gold and hides an equal amount of gold in each of 2 chests, his situation would look something like the picture below.

3 pounds of gold

Chest 1

Chest 2

From the picture above, we can see that each chest would hold half of the three pounds of gold, or $1\frac{1}{2}$ pounds of gold. This problem cannot be modeled by 3÷1/2.

We can also think about this problem without the help of a picture. If a pirate finds 3 pounds of gold, hides an equal amount of gold in each chest, and wants to know how much gold is in each chest, he is really asking, "How much is in each group if 3 is evenly distributed amongst 2 groups?" This is the meaning of
$3 \div 2 = 3 \div \frac{2}{1} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$ not $3 \div \frac{1}{2} = 6$.

3 cups of flour

The whole bag of flour

3 cups of flour

From the picture above, we can see that if 3 cups of flour is $\frac{1}{2}$ the bag, then 6 cups of flour were in the bag to start.

d. This problem is asking "3 cups is $\frac{1}{2}$ of what amount?" which we can represent symbolically like this:

$$\frac{1}{2} \times ? = 3$$

which is equivalent to

$$3 \div \frac{1}{2} = 6$$

This is the "How many in one group?" interpretation of $3 \div \frac{1}{2}$.